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SMALL FATIGUE CRACKS: MECHANICS, MECHANISMS AND ENGINEERING APPLICATIONS

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Small Fatigue Cracks: Mechanics, Mechanisms and Engineering Applications

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Damage-tolerant design and life-prediction methodologies have been practiced for metallic structures for decades, although their application to brittle materials, such as ceramics and intermetallic alloys, still poses particular problems, primarily because of their extreme flaw-sensitivity. Moreover, like metals, they are susceptible to premature failure by cyclic fatigue, which provides a prominent mechanism for subcritical crack growth that further limits life. One specific problem involves the large dependency of growth rates on the applied stress intensity, which necessitates that design is based on the concept of a fatigue threshold, particularly in the presence of small cracks. In this paper, studies on the role of small cracks in influencing thresholds and near-threshold growth rates are described for both brittle and ductile materials. Examples are given from the military engine "High-Cycle Fatigue" initiative, which represents an important problem where information on the behavior of small fatigue cracks is critical.

Keywords: *high-cycle fatigue, fatigue-crack propagation, fatigue thresholds, small cracks, crack-tip shielding, foreign object damage, ceramics, intermetallics, Ti-6Al-4V*

1. Introduction

The problem of small cracks remains one of the most critical, yet least understood, topics in fatigue research, despite the fact that 25 years have elapsed since the so-called anomalous behavior of small fatigue cracks was first reported for age-hardened aluminum alloys by Pearson [1]. Since that time, innumerable papers, reviews and books have been published on the subject [e.g., 2-9] indicating that the "small-crack effect" is associated with the fact that the growth rates of small cracks can exceed those of large cracks at the same *applied* stress-intensity range ΔK, and that small cracks can propagate at applied stress intensities less than the fatigue threshold, ΔK_{TH}, below which large-crack growth is presumed dormant. In general, these effects occur when crack sizes become comparable with (Fig. 1):

- *microstructural size scales*, where biased statistical sampling of the microstructure leads to accelerated crack growth along "weak paths" and local retardation or arrest can occur at microstructural "barriers" such as grain or phase boundaries (a continuum or homogeneity limitation) - *microstructurally-small cracks*,
- *the extent of local inelasticity*, e.g., the plastic-zone size, ahead of the crack tip, where the assumption of small-scale yielding implicit in the use of the stress intensity K is not strictly valid (a linear-elastic fracture mechanics limitation) - *mechanically-small cracks*,

- *the extent of crack-tip shielding*, e.g., crack wedging by crack closure, behind the crack tip, where the reduced role of shielding leads to a higher *local* driving force than the corresponding large crack at the same applied (global) stress intensity (a similitude limitation) - *physically-small cracks*.[1]

Whereas the small-crack effect has always been of prime importance since it can result in gross overestimates of the damage-tolerant life of a structure, the problem has come into greater prominence of late due to (i) the *Aging Aircraft Initiative*, where the residual strength and remaining life of many sections of the fuselage of older aircraft is compromised by numerous small defects i.e., multi-site damage, (ii) the *High-Cycle Fatigue Initiative,* where the failure of blades and disks in aircraft gas-turbine engines results from the rapid propagation of small cracks under high-frequency vibratory loading [9-12], (iii) an increasing emphasis on *durability,* particularly for the economically feasible life of aircraft structures, based on the statistical modeling of small flaw populations [13], and (iv) the potential *structural use of ceramics and intermetallic materials*, especially for engine and aerospace applications, where due to their highly restricted growth-rate curves, i.e., very high Paris-law exponents, design must be based on the concept of a threshold, ideally defined for small cracks [14,15].

Although the behavior of small cracks is now quite well documented for metallic materials, far less information is available for advanced materials. Moreover, a major difficulty with small cracks has always been the difficulty of experimentally determining small-crack data, particularly for design purposes. Accordingly, we describe how *in limited cases* worst-case large-crack data can be used to estimate small-crack growth rates and threshold behavior in both traditional and advanced materials. However, this approach is only viable for cracks larger than the scale of microstructure ("continuum-sized cracks"); it is not appropriate where cracks are of a size that they interact with microstructural features of comparable dimensions (microstructurally-small cracks), due to biased "sampling" of the microstructure.

In general, the behavior of small cracks is seen to be remarkably similar in traditional and advanced materials; specifically the behavior of cracks of a size comparable with the extent of shielding can be readily estimated from worst-case (shielding-corrected) large-crack results. This occurs despite the fact that the salient shielding mechanisms in the fatigue of ceramics and intermetallics are not simply crack closure but additionally involve other mechanisms including *in situ* phase transformations and mostly crack-bridging processes [14,15]. However, when cracks are small compared to microstructural dimensions, this approach cannot work. Moreover, it is shown that the optimal microstructures for small-crack resistance are rarely similar to those optimized for large-crack fracture toughness and fatigue resistance, as shown by recent results in $\alpha+\beta$ [16] and β titanium alloys [17] and for γ-based TiAl alloys [18].

Finally, the problem of foreign-object damage on jet engine fan blades [9-12,19] is shown as an engineering example were the critical conditions for fatigue failure must be defined in the presence of microstructurally-small cracks. In this example, simple worst-case large-crack data cannot be used to estimate small-crack growth rates and threshold behavior. Instead, a modified Kitagawa-Takahashi approach is presented, where the limiting conditions of fatigue failure are determined by the stress-concentration corrected (*S-N*) fatigue limit (at microstructurally-small

[1] There is also the *chemically-small crack* which, unlike a corresponding longer crack, has a local crack-tip environment similar to that of the bulk environment, e.g., due to easier solution renewal [4].

crack sizes) and worst-case large-crack fatigue threshold (at larger, "continuum-sized" crack sizes).

2. Traditional Materials

2.1 Titanium alloys

High-cycle fatigue (HCF) has been identified as one of the prime causes of turbine engine failure in military aircraft. It can result in essentially unpredictable failures due to the propagation of fatigue cracks in blade and disk components under ultrahigh frequency loading, with failure resulting from the rapid growth of small cracks often initiated at microstructural damage caused by fretting or foreign object impacts [10-12]. To prevent HCF failures, design methodologies are required that identify the critical levels of microstructural damage which can lead to such failures. Since the engine components experience high frequency (~1-2 kHz) vibrational loads due to resonant airflow dynamics, often with high mean stresses, even cracks growing at slow per-cycle velocities of ~10^{-10} m/cycle can propagate to failure in a short time period. Consequently, HCF-critical turbine components must be operated below a fatigue threshold, appropriate for flaw sizes typically smaller than ~ 500 μm. Since such small cracks are known to propagate below the large-crack threshold (ΔK_{TH}), design against HCF failure must be based on the notion of a practical small-crack threshold, measured under the representative HCF conditions.

As small-crack testing is tedious and prone to excessive scatter, for design purposes one practical approach is to estimate the physically-small crack thresholds from "worst-case" large-crack threshold tests by attempting to minimize any crack-tip shielding in the crack wake [20]. This approach was evaluated for a Ti-6Al-4V alloy (Fig. 2), α+β processed (solution treated and overaged, STOA[2]) for typical fan blade application. Large through-thickness cracks (>10 mm) in compact-tension specimens were measured at high mean stresses using at, above and below the representative frequencies (~1,000 Hz) using constant-K_{max}/increasing-K_{min} cycling [21] to approach the threshold at extremely high load ratios, i.e., at R (= K_{min}/K_{max}) values approaching unity. Results, shown in Fig. 3 for both (a) constant-R (at R = 0.1 to 0.8) and (b) constant-K_{max}/varying R tests, were found to be independent of frequency for the Ti-6Al-4V alloy in question, over the wide range of 50 to 20,000 Hz for tests in ambient temperature air [20]. Moreover, they indicate that whereas constant-R cycling at R = 0.8 gives a ΔK_{TH} value of 2.6 MPa√m (compared to 4.6 MPa√m at R = 0.1), even lower values are obtained with constant-K_{max} cycling, a worst-case value of ΔK_{TH} = 1.9 MPa√m being achieved with K_{max} = 36.5 MPa√m at R = 0.95. Crack closure could not be detected above R ~ 0.5.

Whereas even lower values can probably be obtained by using higher K_{max} values approaching K_{Ic}, the measured worst-case value of 1.9 MPa√m at R = 0.95 was found to provide a lower bound to thresholds for the onset of the growth of naturally-initiated small cracks (a ~ 45 - 1000 μm), shown in Fig. 4 for the Ti-6Al-4V material under study. Although small-crack growth rates are ~0.5-1 orders of magnitude faster than corresponding large-crack data, no small-crack growth was reported below ΔK = 2.9 MPa√m, i.e., well above the worst-case large-crack threshold [20].

The higher growth rates of the naturally-initiated small-cracks, compared to the "closure-free" large cracks, are presumed to be associated with biased statistical sampling of the microstructure. The natural crack initiation process allows for sampling of a very large area of the specimen for more favorable microstructural orientations with respect to crack initiation and growth, compared

[2] This microstructural condition is also known as a *bimodal* microstructure.

to that of the large cracks which essentially sample an average of all microstructural orientations [e.g., ref. 9].

High-cycle fatigue thus provides a notable example of the importance of small cracks and thresholds for small cracks in traditional materials. Moreover, the approach of defining the type of small cracks, i.e., physically-small cracks in the example of Ti alloy turbine blades, permits the use of worst-case large-crack thresholds (measured under very high R conditions with minimal crack closure) as a feasible and practical solution of obtaining estimates of the small-crack thresholds. However, the use of "worst-case" large-crack thresholds is only feasible in cases where crack sizes are larger than the characteristic microstructural dimensions (and cyclic plastic-zone sizes). In the presence of microstructurally-small cracks, alternative approaches must be sought. One, especially relevant to the high-cycle fatigue problem, involves the Kitagawa-Takahashi approach, as discussed in detail below.

3. Advanced Materials

The importance of small fatigue cracks with advanced materials, such intermetallics and ceramics, is caused principally by the sensitivity of their crack-growth rates to the applied stress intensity; specifically this results in Paris power-law exponents (i.e., m in relationships derived from $da/dN \propto \Delta K^{m}$) often well in excess of 10, compared to metallic materials where typically $m \sim$ 2 to 4 (in the mid-range of growth rates) (Fig. 5) [e.g., ref. 15]. Since the projected damage-tolerant life is proportional to the reciprocal of the applied stress raised to the power of m, a factor of two change in this stress can lead to life projections of a ceramic component (where m can be as high as 15-20 or more) to vary by more than six orders of magnitude. Essentially, because of the high exponents, the life spent in crack propagation in advanced materials is extremely limited; accordingly, rather than basing lifetime calculations on the integration of crack-propagation data, as is typically done with metallic structures, design must be based on the concept of a fatigue threshold, invariably defined for small cracks due to the inherently low tolerance of brittle materials to flaws.

In light of this, an appreciation of the small-crack effect and the documentation of fatigue thresholds for small cracks are, if anything, more important for advanced materials than for traditional alloys. However, there are only a few published results on small fatigue crack-growth behavior in these materials.

3.1 Ceramic materials

3.1.1 Monolithic ceramics

Unlike ductile materials where crack advance is motivated by *intrinsic* cyclic damage mechanisms *ahead* of the tip, e.g., crack-tip blunting and re-sharpening, the cyclic fatigue process in monolithic ceramics, e.g., Si_3N_4, SiC, involves the cyclically-induced suppression of *extrinsic* crack-tip shielding, primarily crack bridging, *behind* the crack tip; at ambient temperatures, the crack-advance mechanism itself is typically identical to that for crack growth under static loading (Fig. 6) [15,22,23]. With this approach, it is apparent that the high Paris exponents m seen in ceramics result primarily from an increased dependency upon the maximum stress intensity, K_{max}, rather than on ΔK *per se*. Indeed, using a modified form of the Paris relationship [15,24]:

$$da/dN \propto \Delta K^{p}\, K^{n}_{max}\,, \qquad (1)$$

where $(n + p) = m$, the exponents n and p are ~36 and 1.9 in a typical ceramic, e.g., *in situ* toughened SiC, respectively [25]; this is to be compared with values of $n = 0.5$ and $p = 3$ for metal fatigue of a nickel-base superalloy [26].

Since there is no apparent intrinsic damage process unique to cyclic loading, fatigue-crack initiation always occurs at pre-existing defects, i.e., unlike crack formation in persistent slip bands in metals, natural crack initiation does not occur. The exception to this is ceramics toughened by *in situ* phase transformations, e.g., MgO partially-stabilized zirconia (Mg-PSZ), where natural initiation can occur and is apparently associated with sites of local transformation [27]. Behavior in both classes of ceramics is briefly described below.

3.1.2 Phase-transforming ceramics

The growth-rate behavior of microstructurally-small surface cracks (<100 μm), naturally initiated on the surface of cantilever bend bars of Mg-PSZ, is compared to the corresponding behavior of large, through-thickness cracks (>3 mm) in compact-tension specimens in Fig. 7 [27]. Small cracks can be seen to propagate below the long-crack threshold, yet individual cracks grow at decreasing growth rates with increasing applied stress intensity, sometimes to arrest. Such behavior is similar to that reported for physically-small cracks in metallic materials where the primary shielding mechanism is crack closure [e.g., 3]; in PSZ, however, the effect results from shielding by transformation toughening [28].

Resistance to crack growth in PSZ is afforded by an *in situ* stress-controlled transformation of tetragonal ZrO_2 precipitates to the monoclinic phase, which results in a zone of compressive material surrounding the crack wake due to an associated dilation of $\varepsilon_T \sim 6\%$ (there is also some degree of shear). Analogous to crack closure in metals, the effective (near-tip) stress intensity, ΔK_{tip}, for large cracks is reduced from the applied value by shielding due to transformation toughening in the crack wake, i.e., $\Delta K_{tip} = K_{max} - K_s$. At steady-state, the shielding stress intensity reaches a value given approximately by [29]:

$$K_s \sim 0.2\, E' \,\varepsilon_T f \sqrt{h}, \tag{2}$$

where E' is the effective elastic modulus in plane stress or plane strain, f is the volume fraction of tetragonal precipitates, and h is the transformation-zone width. The steady-state condition is reached once there is a wake of transformed material extending at least five times h, i.e., the crack has penetrated the zone more than a distance of $5h$ [29]. Since the transformation-zone size is on the order of tens to hundreds of micrometers (depending upon the heat-treated condition of the PSZ), any crack of a length less than roughly five times this dimension would experience a diminished effect of the shielding; this in turn results in an enhanced near-tip stress intensity, compared to a large crack at the same applied K, such that the small cracks can propagate at lower applied K values [27]. The decelerating growth rates result from the mutual competition of the applied K, which increases with increase in crack length, and the shielding K which also increases until a steady-state wake zone has been established [27].

Consistent with this explanation, a “worst-case” crack-growth relationship can be determined for the small-crack data by correcting the large-crack results for the maximum extent of transformation shielding, K_s, computed from Eq. (1). This is shown in Fig. 7(b), where it is clear that the large-crack da/dN vs. ΔK_{tip} relationship, which can be readily measured experimentally, can be utilized as a conservative representation of small-crack behavior [27]. However, this practical solution to the physically-small crack problem is only feasible when i) the prevailing mechanism of shielding is known, and ii) the magnitude of its influence can be either measured or, as in the present case, computed theoretically.

3.1.3 Non phase-transforming ceramics

Although PSZ is perhaps the highest toughness ceramic, the absence of the transformation at elevated temperatures severely limits its use. For high temperature applications at 1000°C or above, silicon nitride and silicon carbide are currently the preferred ceramics. Both materials can be extrinsically toughened above their intrinsic toughness of ~2-3 MPa√m to ~8-10 MPa√m by promoting shielding by interlocking grain bridging in the crack wake [e.g., 25]. This is generally achieved by elongating the grain structure and weakening the grain boundaries (through the presence of the glassy grain-boundary phase) to promote intergranular fracture [30].

Under cyclic loading, such grain bridging is degraded by frictional wear along the boundaries; indeed, as depicted schematically in Fig. 6(b), this is the primary mechanism for fatigue-crack growth in most ceramics at ambient temperatures [15,22,23]. Since the bridging zones can traverse several grains, there is again a potential for small-crack effects where crack sizes are comparable to the size of this zone, i.e., on the order of the grain size.

To date, very few studies [e.g., 31] have focused on the fatigue behavior of cracks of microstructural dimensions in ceramics. As natural initiation cannot occur in these materials, hardness indents have generally been used to initiate cracking such that the typical small cracks studied generally are larger than ~100 μm or so. Two examples are shown in Figs. 8 and 9, respectively, for small surface cracks initiated at indents in cantilever bend samples of an *in situ* toughened monolithic SiC [32] and a SiC-whisker reinforced alumina composite [24]. As before, the small cracks propagate below the large-crack threshold at decreasing rates until they arrest or merge with the large-crack curve. In these cases, the decreasing growth rates are a result of the crack growing out of the residual tensile stress field of the indent. In fact, by determining an effective stress intensity by superposing the stress intensity resulting from this residual field, K_{RD}, and the globally applied stress intensity, the large and small crack data can be brought into correspondence. Similar results have been reported for several ceramics, including Si_3N_4 [33], pyrolytic carbon [34,35], and several other grades of toughened SiC [32].

These examples for brittle ceramics serve to illustrate that for small cracks *larger than* the scale of the microstructure, discrepancies between large and small crack growth rates can be reconciled by a worst-case threshold approach by fully accounting for the various contributions to the local driving force, namely from the globally applied K, any residual stress fields, and the magnitude of the relevant shielding in the crack wake.

3.2 Ordered intermetallic alloys

Due to their complex and ordered crystal structures, intermetallics (like ceramics) generally display only very limited mobile dislocation activity at low homologous temperatures (below their brittle-to-ductile transition temperature), and are thus often highly restricted in ductility and toughness. However, unlike ceramics, they can be toughened both intrinsically and extrinsically, although the former is far more difficult [36]. Whereas intrinsic mechanisms, such as the activation of additional slip systems, do not degrade under cyclic loading, extrinsic toughening mechanisms such as crack bridging can suffer severe cyclic degradation, similar to behavior in ceramics. A notable example of this is ductile-phase reinforced intermetallic-matrix composites, such as β-TiNb-reinforced γ-TiAl, which due to extensive wake bridging by the uncracked ductile phase can display significantly higher toughness (i.e., by a factor of 3 or greater) than the constituent matrix [37]. However, the improvement in crack-growth resistance is far less obvious in fatigue simply because the ductile phase fails prematurely (by fatigue); indeed, the fatigue-crack growth properties are rarely any better than that of the unreinforced matrix.

Nevertheless, as the extrinsic toughening mechanisms act in the crack wake, e.g., in TiNb-TiAl over a bridging zone of several millimeters, fatigue cracks smaller than this dimension will not develop the full magnitude of shielding and as such will behave as physically-small cracks. In addition, the microstructurally-small crack problem is prevalent in certain intermetallic alloys; as discussed below, coarse lamellar microstructures in γ-based TiAl alloys are a good example of this as colony and grain sizes in some alloys can approach and exceed millimeter dimensions [38].

3.2.1 Gamma-based TiAl alloys

Of the various intermetallic alloys considered for structural application, γ-based TiAl have received by far the most attention as possible low-pressure gas-turbine blades. Based on the composition (at.%) of Ti-47Al with small additions of elements such as Nb, Cr, V and B, two microstructures have been most studied (Fig. 10(a),(b)): a *duplex* microstructure, consisting of ~15-40 μm sized equiaxed grains of γ (TiAl) with small amounts of α_2 (Ti_3Al), and a *lamellar* microstructure, consisting of lamellar colonies (several hundred of micrometers in diameter) containing alternating γ/γ and γ/α_2 platelets [38]. In general, duplex structures display better elongation and strength, whereas lamellar structures show better creep resistance, toughness and (large-crack) fatigue properties.

It is well documented that lamellar microstructures possess the far superior fracture toughness and R-curve behavior [e.g., refs. 38,39]. This arises primarily from intra- and inter-lamellar microcracking ahead of the crack tip, which results in the formation of uncracked (shear) ligament bridges [39]. Such bridging degrades somewhat under cyclic loading [40], but is still sufficiently potent to give lamellar structures the far superior fatigue-crack growth resistance [40,41]. As shown by the results for a Ti-47Al-2Nb-2Cr-0.2B (at.%) alloy in Fig. 10(c) [40], at a given applied ΔK, growth rates are up to five orders of magnitude slower, and ΔK_{TH} thresholds some 50% higher, than in the duplex structure. These properties are only realized, however, for large cracks of dimensions larger than the bridging zones. Corresponding results for small surface cracks, with half-surface lengths c ~ 25-300 μm, shown for the same microstructures in Fig. 10(c), indicate that growth rates in the two structures are comparable, although there is clearly more scatter in the data for the coarser lamellar structure [18,40]. At the same applied ΔK levels, the growth rates of the small cracks exceed those of corresponding large cracks by several orders of magnitude; moreover, the small cracks once more propagate below the large-crack thresholds.

If the small-crack data in Fig. 10(c) are compared with the shielding-corrected large-crack results, where the da/dN vs. ΔK_{tip} relationship is derived by measuring the shielding contributions from both crack bridging (K_{br}) (in lamellar structures only) and crack closure (K_{cl}), i.e., $\Delta K_{eff} \equiv \Delta K_{tip} = (K_{max} - K_{br}) - K_{cl}$, the large and small crack data come into normalization (Fig. 11), but only for the duplex microstructure [40]. In this much finer-scale microstructure, the small cracks studied are only comparable in size to the wake shielding zones; thus, as for similar physically-small cracks in the metallic and ceramic materials, their behavior can be described by shielding-corrected large-crack data. In contrast, small cracks in the coarser lamellar microstructure are seen grow at applied stress intensities *below* the large-crack ΔK_{TH} (Fig. 11(b)), such that a limited equilibrium shielding zone cannot be the sole cause of the small-crack effect in this structure [18]. In fact, cracks in this structure are comparable to microstructural dimensions as average colony sizes (~145 μm) are on the same order of magnitude as the cracks under study (c ~ 25 - 300 μm). The influence of the coarser lamellar structure on small-crack behavior is apparent in Fig. 11(b), where the small-crack data are divided into cracks with initial surface crack lengths smaller and larger then the average colony size [18]. From this comparison, it is apparent that all the small cracks that were observed to propagate below the "shielding-corrected" large-crack threshold had initial crack lengths smaller than the average colony size, while no small cracks with initial crack lengths larger than the average colony size grew below this threshold. Although using the colony size as the critical microstructural dimension is somewhat arbitrary, it does show that for cracks contained within one or two lamellar colonies, the growth behavior is quite different from that observed when a larger crack can "sample" the continuum microstructure.

Thus, duplex microstructures may offer far better HCF properties than lamellar microstructures for use in many applications such as turbine blades, despite the fact that the latter structure displays markedly higher toughness and large-crack growth resistance. In addition to having higher strength and ductility, the definition of a worst-case (shielding-corrected) large-crack threshold, for both large and relevant small cracks, appears to be feasible in the duplex structure, whereas such an approach is far less certain for lamellar structures where relevant crack sizes are microstructurally small.

4. Engineering Applications

An essential theme in this paper has been the use of worst-case large-crack thresholds to estimate the near-threshold behavior of "continuum-sized" small cracks. However, it has also been noted that this approach will not work where crack sizes approach microstructural dimensions, i.e., in the presence of microstructurally-small cracks. An excellent example of this is the problem of foreign-object damage (FOD) in inducing high-cycle fatigue (HCF) failures of fan blades in aircraft gas-turbine engines.

Because of an increasing number of recent incidents of HCF-related military engine failures, currently used HCF design methodologies, based on stress-life (*S-N*) curves and the Goodman diagram, are being re-evaluated [10-12]. Both FOD and fretting, particularly in the blade dovetail/disk sections, are identified as critical problems [11,42], which can lead to premature fatigue crack initiation and growth; this in turn can result in seemingly unpredictable *in service* failures, due to the high-frequency vibratory loading (>1 kHz) involved [11]. In light of this, design against HCF based on the damage-tolerant concept of a fatigue-crack growth threshold (ΔK_{TH}) for no crack growth would appear to offer a preferred approach; however, such thresholds must reflect representative HCF conditions of small crack sizes, high frequencies and high mean stress levels (depending on the blade span location) [10-12,43].

As a basis for such an approach, studies have been focused on the role of FOD in affecting the initiation and early growth of small surface fatigue cracks in the STOA Ti-6Al-4V alloy (Fig. 2) under HCF conditions. The prime effect of FOD in markedly lowering resistance to HCF was found to be due to earlier crack initiation. Specifically, premature crack initiation and subsequent near-threshold crack growth is promoted by the stress concentration associated with the FOD indentation [9,19,42,44,45] and the presence of small (micro)cracks [19] in the damaged zone; in addition, residual stress gradients [9,19,45] and microstructural changes [9,19] due to FOD-induced plastic deformation can play an important role.

To simulate FOD, the flat surface of fatigue test specimens was subjected to high-velocity (300 m/s) impacts of 3.2 mm dia. hardened steel spheres [9,19]; a typical damage site from a normal (90°) ballistic impact is shown in Fig. 12. Characteristic of such damage in titanium alloys

[46,47], intense so-called adiabatic shear bands can be seen at the surface of the impact crater; moreover, a pronounced pile-up of material, some of it detached, is evident at the crater rim. However, of particular note is that plastic flow of material at the crater rim results in multiple notches and the occurrence of microcracks (insert in Fig. 13). These microcracks are quite small, i.e., between ~2 to 50 μm in surface length, but when favorably oriented with respect to the subsequently applied fatigue stresses, they clearly provided the prime sites for premature crack nucleation and growth in high-cycle fatigue (Fig. 13).

Because of such phenomena, the *S-N* fatigue life of undamaged smooth-bar specimens is significantly affected by prior high-velocity impact damage (Fig. 14). Shown in this figure are the surface crack lengths of the FOD-induced microcracks, which are all comparable with microstructural size-scales. Although the presence of these small cracks represents the primary feature responsible for the FOD-induced reduction in fatigue resistance, tensile residual stresses (estimated in ref. [48]), the highly deformed microstructure beneath the indent [9,19], and the stress-concentration factors associated with the indent ($k_t \sim 1.25$) are all contributing factors.

Thresholds for fatigue-crack growth and the subsequent near-threshold fatigue-crack growth rates for the extension of the small (micro)cracks were measured [9] on all FOD-damaged samples; results are compared in Fig. 15 with previous results on this alloy for the behavior of naturally-initiated small (~45 - 1000 μm) cracks [16] and through-thickness large (>5 mm) cracks [20] in undamaged material. The small crack growth-rate data are shown as a function of surface crack length, $2c$, and of the applied stress-intensity range. Approximate local stress intensities for small cracks at the indentation rim were calculated from Lukáš' solution [49] for small cracks at notches (which includes indentation geometry and stress concentration effects), in terms of the crack depth a, indentation radius ρ, stress range $\Delta\sigma$, and elastic stress-concentration factor k_t = 1.25 [50]:

$$\Delta K = \frac{0.7\,k_t}{\sqrt{1+4.5\left(a/\rho\right)}}\,\Delta\sigma\sqrt{\pi\,a}\;. \tag{3}$$

The factor of 0.7 is based on the stress-intensity boundary correction and the crack-shape correction factors [51]. It should be noted that Eq. 3 does not take into account the presence of residual stresses in the vicinity of the indents. To the first approximation, however, they do not change the value of the stress-intensity range (ΔK); they merely affect the mean stress and hence alter the local load ratio. Moreover, their effect of may not be that critical, as *in situ* synchronous X-ray micro-diffraction measurements [52], coupled with quasi-static numerical modeling [48], show a significant relaxation in their magnitude on fatigue loading, even within a few cycles.

The FOD-initiated small-crack growth rates in Fig. 15 are positioned roughly between the large-crack data (as a lower bound) and naturally-initiated crack data (as an upper bound). Typical for the small-crack effect [e.g., ref. 7], most small-crack growth rates are roughly an order of magnitude faster than corresponding large-crack results at near-threshold levels. Above a ΔK of ~10 MPa√m, however, the large and small crack results tend to merge, consistent with the development of a steady-state shielding zone in the wake of the small cracks.

As discussed previously, the large-crack results in Fig. 15 were determined up to the highest load ratios ($R \sim 0.1 – 0.95$) under conditions (constant-K_{max}/increasing-K_{min}) chosen to minimize the effect of crack closure; the threshold of ~1.9 MPa√m at R = 0.95 was thus considered to be a "worst-case threshold" for large or physically-small cracks of dimensions large compared to the scale of the microstructure, i.e., for "continuum-sized" cracks [20]. Of significance in the present results is that the smallest FOD-initiated cracks have dimensions of ~2 to 25 μm, which are comparable with microstructural size-scales in Ti-6Al-4V. These microstructurally-small cracks propagate at stress intensities well below this "worst-case" threshold, specifically at applied stress intensities as low as ΔK ~1 MPa√m.[3] The implication of this result is that although there is a definitive lower-bound ΔK_{TH} threshold (~1.9 MPa√m) for "continuum-sized" cracks in the STOA Ti-6Al-4V alloy, when crack sizes approach microstructural size-scales, as in the case of the FOD-induced microcracks, crack growth is possible at applied stress intensities considerably less than this threshold, presumably due to biased sampling of the "weak links" in the microstructure. However, by coupling the notion of the worst-case threshold stress intensity with the fatigue limit, as in the Kitagawa-Takahashi diagram [53], an alternative formulation can be developed for defining the limiting conditions for HCF and FOD-related damage, as described below.

As an alternative HCF-design approach against FOD, which takes into account the presence of microstructurally-small cracks [19], Fig. 16 shows the influence of crack size on the worst-case fatigue thresholds in the form of a modified Kitagawa-Takahashi diagram [53], where the stress range for a corresponding crack-growth threshold condition (da/dN = 10^{-11}-10^{-10} m/cycle) is plotted versus the crack length. According to the Kitagawa-Takahashi approach (solid lines), the stress range $\Delta\sigma$ for crack arrest is defined by the 10^7-cycle fatigue limit ($\Delta\sigma_{HCF}$ = 450 MPa) with the fatigue-crack growth threshold (ΔK_{TH}) measured on "continuum-sized" cracks. El Haddad *et al*. [54] empirically quantified this approach by introducing a constant (intrinsic) crack length $2c_o$, such that the stress intensity is defined as $Y\Delta\sigma\sqrt{\pi\left(2c+2c_o\right)}$, where Y is the geometry factor (curved dashed line). The present results, plotted in this format in Fig. 16, show that the threshold conditions for crack growth from FOD-induced microstructurally-small cracks can be defined, in terms of *stress concentration corrected* stress ranges, from the "El Haddad" line in the Kitagawa-Takahashi diagram, provided the limiting conditions are described in terms of the 10^7-cycle (smooth-bar) fatigue limit (at microstructurally-small crack sizes) and worst-case large-crack fatigue threshold (at larger, "continuum-sized", crack sizes).

5. Concluding Remarks

The question of small fatigue cracks in metallic components remains a critical issue in the application of damage/fracture mechanics to the design, durability and damage tolerance of many structures, particularly for aircraft and gas-turbine applications. This has been highlighted for the problem of military engine high-cycle fatigue, especially in the presence of foreign-object damage. The problem of small fatigue cracks is similar in ceramic and intermetallic materials, although the micro-mechanisms of crack-tip damage and shielding are quite different. In all cases, failure to address this issue can lead to severely non-conservative lifetime predictions. Due to the difficulty of obtaining small-crack data, particularly at crucial near-threshold levels, worst-case large-crack data, where the effect of the predominant shielding mechanisms on the *local* driving force has been minimized or accounted for, can be used to estimate small-crack growth rates and threshold values, provided the cracks are small with respect to the extent of the wake shielding, i.e., physically small. Whereas this approach is feasible for most structural materials for crack sizes down to ~50-100 μm, it will not work where relevant crack sizes approach the characteristic microstructural dimensions. Due to their biased sampling of the microstructure, thresholds for the

[3] It is believed that a ΔK of 1 MPa√m is the lowest stress-intensity range ever measured for crack growth in Ti-6Al-4V, representing a ΔK value roughly a factor of two smaller than the lowest measured fatigue threshold for a "continuum-sized" crack.

onset of cracking and the subsequent near-threshold crack-growth rates for such microstructurally-small cracks must be determined as a function of crack size. In these instances, a modified Kitagawa-Takahashi approach, with the limiting conditions of the *S/N* fatigue limit at microstructurally-small crack sizes and the worst-case, large-crack, fatigue threshold at larger, "continuum-sized", crack sizes, provides a viable alternative.

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